Name: Solution Key

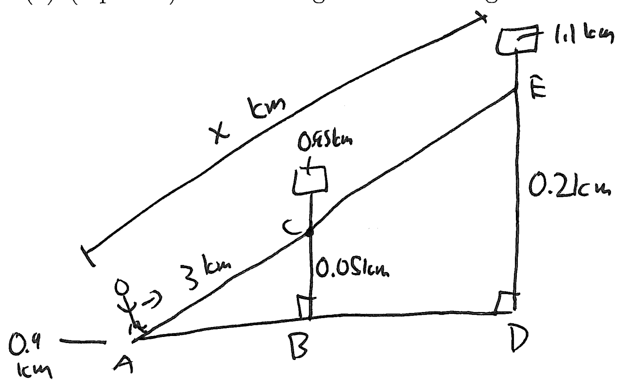
MA 202 EXAM 3: April 10, 2018

Instructions: The following exam has 90 possible points. The point value of each question is stated explicitly. No books or notes may be used on this exam. Please **write legibly** and keep your paper as organized as possible. You **may not** use a calculator on this exam. **Show all your work!** Answers without explanation will not receive full credit. Use complete sentences where appropriate. If you have any questions, be sure to ask. Good luck!

Question	Points	Score
1	10	
2	12	
3	10	
4	10	
5	8	
6	15	
7	15	
8	10	
Bonus	5	
Total:	90	

1. (10 points) Lavender began walking up a hill at a spot where the elevation is 0.9 km. After she walked 3 km, she saw a sign giving the elevation as 0.95 km. She keeps walking until she reaches an elevation of 1.1 km.

(a) (3 points) Draw a diagram illustrating this scenario with all distances clearly labeled.



(b) (3 points) Find two similar triangles in your diagram above and justify why they are similar.

$\triangle ABC$ and $\triangle ADE$ are similar by AA similarity since they share angle A and both angle B and angle D are right angles.

(c) (4 points) How far does she walk in total?

Set up a proportion from $\triangle ABC \sim \triangle ADE$:

$$\frac{3 \text{ km}}{x \text{ km}} = \frac{0.05 \text{ km}}{0.2 \text{ km}}$$

$$3 \cdot \frac{0.2}{0.05} = x \text{ km}$$

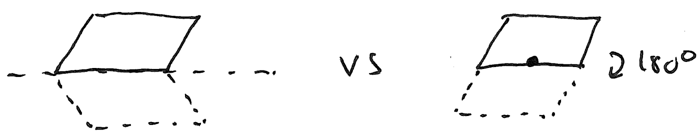
$$3 \cdot 4 = x$$

$$\boxed{x = 12 \text{ km}}$$

2. (12 points) Determine whether each of the following statements is true or false. Justify your response. Each part is worth 2 points.

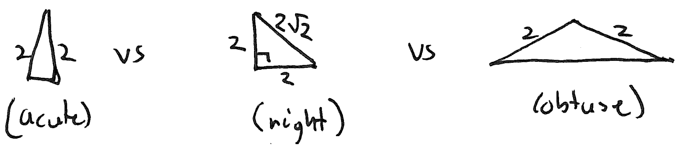
(a) The image of a reflection of a parallelogram across one of its sides is the same as the image of rotating the parallelogram 180° around the midpoint of that side.

False.



(b) All isosceles triangles with a side of 2 meters are similar.

False.

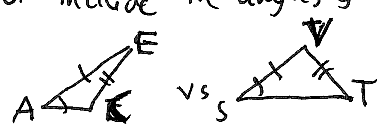


(c) Translating a quadrilateral by 2 units to the right and then rotating it by 45° clockwise around a vertex has the same effect as first rotating and then translating it.

True. Translating any shape and then rotating about one of its points can be done in either order.

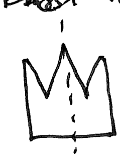
(d) If $\angle A \cong \angle S$, $CE \cong TV$, and $EA \cong VS$, then $\Delta ACE \cong \Delta STV$.

False. The two sides given do not include the angles given, and SSA is not a congruence theorem.



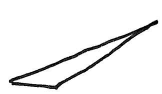
(e) If two heptagons are congruent and each has a line of symmetry, then they are both regular heptagons.

False. Heptagons may have a ~~~~ line of symmetry without being regular.

e.g. 

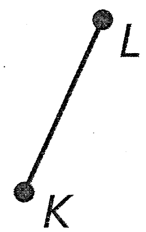
(f) A triangle has 6 lines of symmetry.

False.



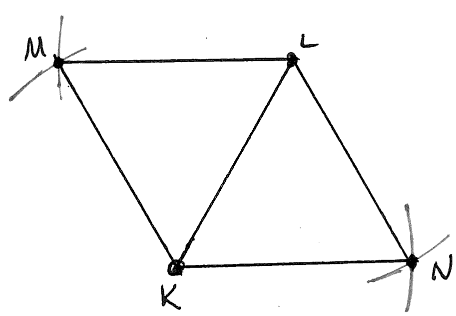
this scalene triangle has 0 lines of symmetry

3. (10 points) (a) (4 points) Describe a procedure using compass and straightedge to construct a rhombus whose sides are the same length as the line KL below. (Hint: such a rhombus can be formed from equilateral triangles.)



1) Set the compass to the length of KL.
2) Place the point of the compass at K and draw an arc on each side of LK. Repeat at L.
3) Mark the intersections of these arcs M and N.
4) Connect K and L to M and N with the straightedge.

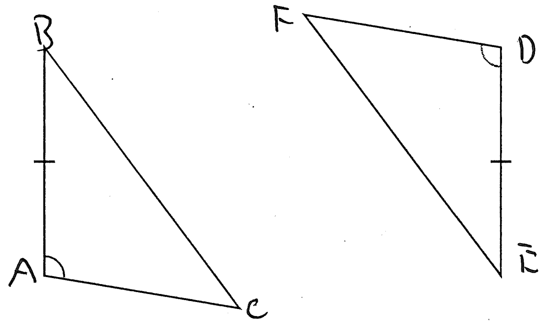
(b) (2 points) Use a compass and straightedge to carry out your construction from part (a).



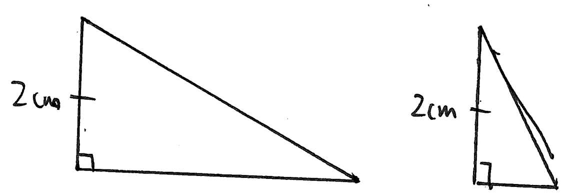
(c) (4 points) Justify your technique.

By construction, each point of intersection of the arcs is distance $\overline{KL}$ from K and L. So $KM \cong KN \cong LM \cong LN \cong KL$, hence quadrilateral KMLN is a rhombus whose sides are the same length as the line KL.

4. (10 points) Consider the pair of triangles below.



(a) (6 points) There is not enough information above to determine that the two triangles are congruent. Confirm this by using your ruler to sketch two triangles that satisfy the given properties (i.e. one angle and one side length of one triangle are congruent to one angle and one side length of the other) but are *not* congruent. Label the necessary information.



(b) (4 points) Identify an additional piece of information needed to show that the triangles are congruent. Explain your reasoning.

1) Knowing that $\angle C \cong \angle F$ or $\angle E \cong \angle B$ is sufficient since then we can use the ASA congruence theorem.

2) Knowing that $AC \cong DF$ is also sufficient since then we can use the SAS congruence theorem.

5. (8 points) A toy erumpent is 16.8 cm tall and is made using the scale $\frac{4}{5}$ cm : 2 ft.

(a) (4 points) Set up and solve a proportion to determine the height of the actual erumpent.

Let h be the height of the erumpent. Then

$$\frac{16.8 \text{ cm}}{h \text{ ft}} = \frac{4/5 \text{ cm}}{2 \text{ ft}}$$

$$33.6 = \frac{4}{5}h$$

$$h = \frac{5}{4} \cdot 33.6 \text{ ft}$$

$$= 5 \cdot 8.4 \text{ ft}$$

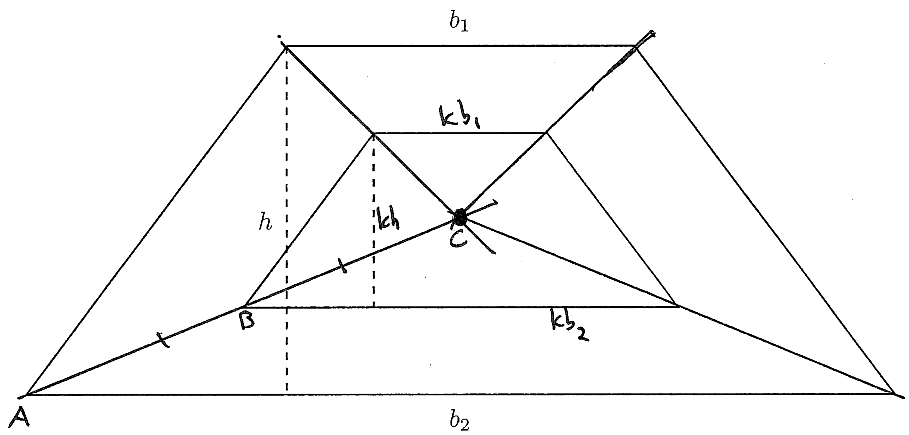
$$= \boxed{42 \text{ ft}}$$

(b) (4 points) Find a scale factor (with common units) describing the relation between the height of the toy erumpent and the actual erumpent. You may use the conversion 1 cm ≈ 0.4 in.

1) We know the scale is $\frac{4}{5}$ cm : 2ft. Converting both sides to inches gives $0.8 \text{ cm} \cdot \frac{0.4 \text{ in}}{1 \text{ cm}} : 2 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}}$ or $0.32 : 24$ or $1 : 100 \cdot \frac{24}{32}$ or $1:75$.

2) We know the toy model is 16.8 cm tall and the erumpent is 42 ft tall. Converting to inches gives $16.8 \text{ cm} \cdot 0.4 \frac{\text{in}}{\text{cm}} : 42 \text{ ft} \cdot \frac{12 \text{ in}}{\text{ft}}$ or $6.72 : 504$

6. (15 points) A scale factor of $k < 1$ is used to reduce a trapezoid as depicted below. The dimensions of the original figure are labeled.



(a) (3 points) Label the dimensions (in terms of k) of the image.

See diagram.

(b) (4 points) If the *image* in the dilation above has a perimeter of 34cm, determine the perimeter of the *original figure* in terms of k. (Your answer should not involve any variable other than k.)

$34_{cm} = P_{image} = k \cdot P_{orig}$, so $\boxed{P_{orig} = \frac{34}{k} \text{ cm}}$

(c) (4 points) Use a straightedge to find the center of dilation. Mark the center of dilation C.

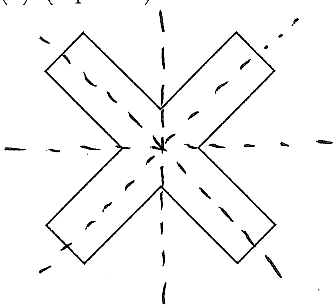
Draw lines connecting corresponding points.

(d) (4 points) Estimate the scale factor k. Clearly indicate (or mark on the figure) the distances you use to determine the scale factor.

$AB \cong AC$, so $\frac{BC}{AC} = \frac{1}{2} = k$.

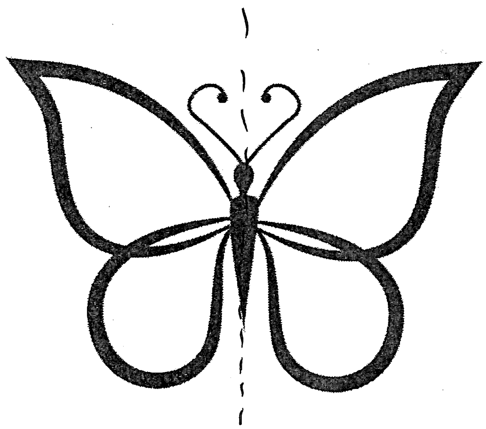
7. (10 points) For each of the figures below, list all types of symmetry that the figure possesses. If it has no symmetries, say that it is asymmetrical.

(a) (4 points)



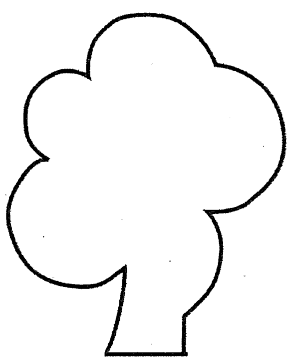
There are 4 lines of symmetry (pictured).
The figure also has rotational symmetry by 90° or 180°.

(b) (3 points)



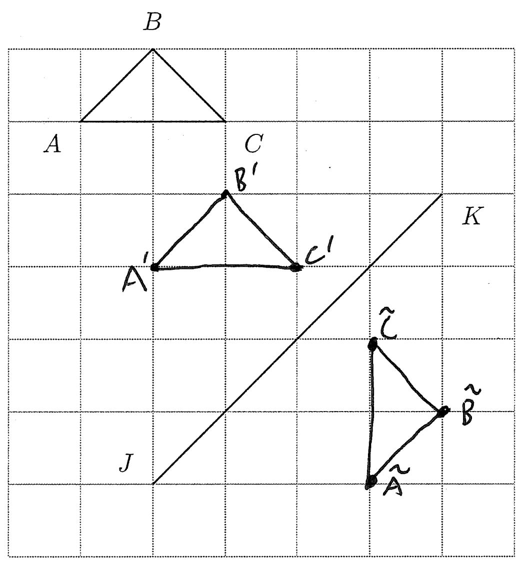
The figure has only vertical symmetry.

(c) (3 points)



The figure is asymmetrical.

8. (15 points) The grid below contains a triangle, ΔABC, and a line segment, $\overline{JK}$.



(a) (5 points) Translate ΔABC two units down and one unit right. Label the image under translation $\Delta A'B'C'$.

See grid.

(b) (6 points) Reflect the image $\Delta A'B'C'$ over line $\overline{JK}$. Label the image under reflection $\Delta\tilde{A}\tilde{B}\tilde{C}$.

See grid.

(c) (4 points) Is the transformation composed of the translation in (a) followed by the reflection in (b) a glide reflection? Why or why not?

It is not. A glide reflection is comprised of a translation which is in a direction parallel to the line of reflection and this translation is not.

- You could also see this by doing the actions in the other order. You will get a figure much further from JK, whereas in a glide reflection the translation and reflection can be done in either order to get the same image.

9. (5 points) Bonus.

(a) (2 points) Are there any uppercase English letters which have a line of symmetry that is **not** horizontal or vertical? If so, draw one.

Possibilities: O Q X

only if 45° cw \

Only if right angles

(b) (3 points) Consider the following three transformations of a rectangle:

- Rotate the rectangle by 45 degrees around its upper left vertex.
- Reflect the rectangle across one of its diagonals.
- Translate the rectangle by 4 units vertically.

Order these three transformations in increasing order by the number of points that do **not** move in the transformation.

Least points not moving	Translate	(0 points)
↓	Rotate	(1 point)
most points not moving	Reflect	(all points on the line of reflection)